Experiment 2-1. What’s Happening Between Charges?

– Coulomb's Law –

Caution: Since 10,000 V penetrates about 10 mm, a high voltage is difficult to supply within a short distance. Especially for high humidity condition, the voltage leaks into the air, thus even applying it could be difficult. In other words, if the power of current supplier is not enough, it could be difficult to achieve the high voltage because of the leak of very low current into the air. If the distance between electrodes is too short, the spark occurs in the space between them. Since it can affect digital scales, you should prevent it. It can also give an electric shock, so be careful to manage + electrode of a power supply. However, it's not so dangerous because the current is weak. The maximum current of the power supply is about 2mA.

Electric charges are the sources of electric phenomena and a characteristic of a particle like the mass. An electric phenomenon means the interaction (force) between charge and charge. Coulomb's law is the quantitative relation between the electric force and the charges \( q_1 \) and \( q_2 \), given by \( F \propto \frac{q_1 q_2}{r^2} \), which shows the electric force is inversely proportional to the square of the separation \( r \) between the charges. If two charges \( q_1 \) and \( q_2 \) are of same sign the force is repulsive, and if the charges are of opposite sign the force is attractive. It is proposed by Joseph Priestley, Henry Cavendish, Lord Stanhope and others. Charles Augustin Coulomb proved it in 1785, which brought much discussion, and then it has been accepted generally from around 1825. Coulomb used his famous torsion balance at the experiment. He was possible to experiment the dependence of the electric force on the distance because he had found the magnitude of a torsion depends on the twist angle at the torsion balance just a year ago, 1784.
The left picture illustrates the torsion balance Coulomb used in 1785. He measured repulsive forces between charges of same sign for 3 different separations, then showed the electric force follows the inverse square law. The right picture illustrates that the repulsive force between two charged spheres of same sign is balanced by the torsion. Coulomb changed the distance between the spheres by twisting the torsion string by hand which changed the torsion. This method cannot be applied for the case of two oppositely charged spheres. Why? But Coulomb also confirmed the inverse square law for the attractive force, where he used the period of oscillation. Guess how he measured the inverse square law.

In this experiment, we examine the forces between two charged conducting plates and confirm Coulomb’s law indirectly.

### Outline of Experiment

- See how charged objects interact to know why they interact.

  - How can you charge an object?

  - Measure the force acting on a charged object. What method would be good?
Experimental Method

1. The distance between the conducting plates are measured by a micrometer or a ruler. The equipment setup is shown in picture 1. The switch of the power must be in off position. And before turning on, minimize the voltage controller. More important thing is that two conducting plates should not be in contact. Begin experiment with 5mm gap between conducting plates as shown in picture 2. This gap standard can be changed according to the experimenter and situation.

2. The digital scale must be set on the flat place and voltage meter of power supply must be set on 0 when the power supply is off. Insert an insulating plate between conducting plates. Even if there is no insulating plate it is regarded that the insulating plate made of air (~vacuum) is inserted.

   You can make the weight of things placed on the digital scale be displayed as 0 by pushing the “ 규정기” (container) button on it, so that it is easy to see the change since it displays only the increase or decrease in weight.

3. Before connecting output connectors to condenser plates, ground both plates by facing output connectors connected to each condenser plates. Then, connect output connectors to condenser plates, (+) with the upper plate and (-) with the lower plate. If you want to ground condenser plates, you can use the outlet having the grounding pin.

4. Measure the distance between two plates using the micrometer and confirm its uniformity with your eyes. If the plates are curved or the distance is not uniform, make it constant. When you lower the upper plate, it must meet the lower plate perfectly.
To apply voltage to conducting plates, check the scale reading is 0. Then turn on the switch of the power supply and increase the voltage gradually to the wanted value. The scale's value will be changed while the voltage is changing. If too high voltage is applied to conducting plates, arc discharge can occur. Experimenter should perform experiment giving attention to the danger of get shocked. Even if the change doesn't appear immediately, it will come after a while. (This variation has "-" value which means the weight is reduced. If the force is repulsive, the value will be increased.)

(6) To change the distance between plates, minimize the voltage and remove all charges by shorting (-) connector to (+) connector, and then change the distance and continue the measurement in the same way above.

(7) In the measurement 1, check the effect on scale mass where the voltage is constant and the distance is being changed. In the measurement 2, check the effect on scale mass where the distance is constant and the voltage is being changed. see (5) and (6) to see the way changing voltage and distance. In measurement 1 experiment 1 can be conducted with changing initial voltage, and experiment 2 can be conducted with changing initial distance. In these processes record fixed physical quantity and variant physical quantity(ex: In measurement 1, voltage is the fixed physical quantity where distance is the variant physical quantity.)

(8) Repeat the same experiments for other dielectric plates in measurement 3.

Mind that the distance between conducting plates should fit the thickness of dielectric plates so that the error is small. Therefore the experiment conducted with distance change is not valid, and it will be a good experiment to check the magnitude of electrical force according to voltage difference with or without dielectric plates.

(Tip. To fit the distance between the conducting plates to the thickness of dielectric plate, insert insulating plate between conducting plates separated by almost the thickness of dielectric plate by eyes. Then use micrometer to change the distance between conducting plates. The scale reading will be changed as the distance alters. But there will be moment where the scale reading stays constant even the distance between plates is changing. The exact distance at that moment can be thought as the thickness of dielectric plate.

(9) Extend your understanding about capacitors by composing several dielectric plates as series or parallel. The experiment result should be interpreted with high conscience to the fact that the dielectric plates are square.
- Experiment Note (Example)

Dielectric material → [ air (~vacuum) , glass, acrylic plate, Teflon plate ]

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<thead>
<tr>
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<th>Distance, m</th>
<th>applied voltage, V</th>
<th>Scale mass, g</th>
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<td>Exp. 2</td>
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We connect two conducting plates which can be called condensers with a high voltage power supply. When a voltage is applied, charges of same magnitude \( q \) but of opposite sign are produced on each plate according to the geometrical structure, which satisfy

\[
C = \varepsilon_0 \frac{A}{d}, \quad q = CV
\]

where \( \varepsilon_0 \) represents the permittivity, \( A \) the area of a plate and \( d \) the distance between plates.

As shown by picture 2, the electric field is uniform in the middle but is curled in the edges. So we know the electric field is not uniform.

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**Figure 1.** Charges accumulated on a parallel plate capacitor

**Figure 2.** Electric field near the edges of the conducting plates is not uniform.
Let's examine the force between two condenser plates by understanding the energy stored in the capacitor.

There are many capacitors of various sizes and shapes. Two separated conductors are called the conducting plates whatever they look like. A parallel plate capacitor consists of two parallel conducting plates of area $A$ separated by $d$. When it is charged the conducting plates have charges of same magnitude but of opposite sign, that is $+q$ and $-q$ respectively. Here, the charge of a capacitor means the absolute value of charges of plates $q$. ($q$ is not total charge of two plates. Notice the total charge is 0.) Since the plates are conductors, they are equipotent. The electric potential is constant in a conducting plate. But there is the potential difference between two plates. By customary, the absolute value of a potential is just $V$ not $\Delta V$. The charge $q$ of a capacitor is proportional to the potential $V$.

$$q = CV$$

The proportionality constant $C$ which is determined by the geometrical shape of the conducting plate is called the capacitance.

When a voltage is applied on two conducting plates by a high voltage power supply, charges of $+q$ and $-q$ are opposite each other so that a potential difference is generated. The higher the voltage is, the more the opposite charges are, so the more charges are accumulated. Since the conducting plates are metals, there is no potential difference within a plate, therefore the charges are uniformly distributed. If we neglect the curling of electric potential in the edges, the electric field is uniform between two plates. So the energy density $u$, the amount of potential energy per unit volume, must also be uniform. $u$ can be derived by dividing the total potential energy by the volume $Ad$ between the plates.

$$u = \frac{U}{Ad}$$

Here, $U = \frac{1}{2}CV^2$, $C=\varepsilon_0 A/d$, hence the above equation becomes

$$u = \varepsilon_0 (V/d)^2/2$$

Now, we derive the force the lower plate experiences. Suppose the potential difference between two plates is $V$. Neglecting the edge effect, the uniform electric field is produced. (Figure 2)

$$E = \frac{V}{d}$$

As a result, the charge $q$ in the lower plate experiences the force below.

$$F = qE' = qV/2d$$

($E'$ is the electric field by the upper plate the lower plate feels.)
But, since \( q = CV \),

\[ F = CV^2/2d = \varepsilon_0 AV^2/2d^2 \]

which is applied to the lower condenser plate.

When two capacitors of capacitances \( C_1 \) and \( C_2 \) are connected in series or row, the total capacitance is given by

\[
\frac{1}{C_{\text{tot}}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(series)}
\]

\[ C_{\text{tot}} = C_1 + C_2 \quad \text{(parallel)} \]

Advice (about unit)

The following relation is useful to treat units.

\([\text{Coulomb} \times \text{Voltage}] = [\text{Newton} \times \text{meter}] \) (because \( W = qV =Fd \))

The measured quantities are masses so the gravitational constant \( g \) must be divided.

References

1. The existing experiments have created or produced charges using such as like frictional electricity which is produced by rubbing an ebonite bar or some plastics with a fur. Then they transfer them to two insulated spheres. They measure the angle between two spheres using the torsion balance.

2. Since they measure very tiny forces (It is difficult to charge large amounts), they are used sensitive devices. But the static electricity affects all the things near it, it is a bit impossible to be accurate. They sometimes use the Van de Graaff generator to get a high voltage.

3. A digital scale uses the strain gauge sensor so that the distance between the conducting plates doesn't vary not like in a spring scale. In a strain gauge, the pressure acting on the connector sensor is changed, so the experiment can be done while maintaining the distance.
The capacitance between two condenser plates

V is a voltage between two conducting plates and C is a capacitance. They are important factors having characteristics of a condenser.

The unit of C is farad F, which is very large quantity so we normally use µF or pF. 1pF = 1E-12 F

The larger the area of the plates is, the more charges are accumulated as you can guess. And the farther the distance is, the less charges are because the electric force becomes more weaker.

When a voltage of 10,000V is applied to two conducting plates of 200mm×200mm with 10mm separation, the capacitance is 35.4E-12F= 35.4pF. The charge in a plate is 35.4E-12 ×1E4 = 35.4E-8C. (Edge effects are ignored.)

When two charges +q and -q are separated by r, they exert force on each other given by

\[ F = \frac{kq_1q_2}{r^2} \]

Here, k is a constant. This relation is called Coulomb's law in accordance with the name of Charles Augustus Coulomb who firstly derived. (Surprisingly it is the same form with the gravitational force between two particles of mass m1 and m2 with separation r.) Like the gravitational constant G, k is called the electrostatic constant. The gravity is always attractive, but the electrostatic force can be attractive or repulsive according to the charges' signs. It is because the mass has only one kind but the charge has two kinds.

By historical reason and its possibility of reducing many equations, the electrostatic constant is normally used as 1/4πε₀. Then the Coulomb's law is written as

\[ F = \frac{q_1q_2}{4\pi\varepsilon_0 r^3} \]
Here, the constant has the value of

\[ \frac{1}{4\pi \varepsilon_0} = 8.99 \times 10^9 \text{Nm}^2/\text{C}^2 \]

The constant \( \varepsilon_0 \) is called the dielectric constant, which is given by

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2 \]